Robotics files

- ↳ Intro chapter
- ↳ Homogeneous transformation
- ↳ Robot motion analysis.
- ↳ Problems
- ↳ Previous (midterm & final) exams

Thanks to : Marwa saeed

②

Ch 1 L1 Mano

1. Define a Robot

1. a robot is a mechanical device with links and joints, guided by sensors, driven by actuators and Controlled through a programmed Software, to handle and manipulate parts, materials, tools, and devices for performing various tasks in variety of work environments.

+(8) ch2) P(23)

Automation and Robotics

- For the operation and Control of production or manufacture the mechanical, electrical, electronics and computer based systems are integrated to form technology called automation

انواع ال automation ??

2. • Compare between Hard automation and Soft autom

Features	Hard Automation	Soft Automation
1. cost effective	good at very high production volume لو محتاج كميه انتاج كبيره	good for moderate جدا production volume انتاج معين
2. flexibility التغير	Limited	High
3. Life cycle	To be for Longer period	For short and medium period
4. Batch production	Not Suitable	High Suitable
5. Control through SW	Not possible	Easily possible
6. Efficiency of the operation	Comparably high هنا اعلى مقارنه بال soft	Equally High

7. obsolescence of machine مشكلة لما الآلة تقدم	Happens with change in model for which the part is manufactured	Does not happen because the software can be changes
8. Examples	- Automatic Machine - Special purpose machine - Transfer Lines - Machines not Controlled through Sw	- CNC machines - Robots and programmable machines

2 **Robot Advantages**

1. Environment safty

يمكن للروبوت العمل فى بيئات عمل صعبة دون التأثر مثل Light variation، noise polution، Air polution والتى لا تناسب الإنسان للعمل فيها

2. productivity parameter الإنتاجية

الوقت اللازم لإنتاج ~~العمليات~~ المنتجات وإجراء العمليات باستخدام الـ Robot أسرع من الانسان وبالتالى تزداد الإنتاجية.

3. Unit cost in the long run and batch

batches استخدام الـ Robot لإنتاج أجزاء من صورة و يمكنه إنتاج نفس المنتج لمدة طويلة

4. Accuracy, repeatability and work quality of new

استخدام الـ Robot فى العمليات التى تحتاج دقة و تكرار بشكل دائم حيث ينتجها بجودة عالية

(5) Simulation multiplicity, it can work with multiple stimuli
manipulators are capable of performing tasks of many stimuli at a given instance.

(6) Advanced technologies can be associated with Robot such as Sensors & Camas
التي تجعل ال Robot يحاكي الإنسان في الرؤية.
والتقاط الأشياء التي يستطيع أن يراها.

Disadvantages

(1) Cost constraint in investment
تكلفة شراء الروبوت والتدريب وتشغيله وصيانته
تجعل اتخاذ قرار شراء روبوت

(2) Decision intelligence
ال Robot لا يستطيع ان يفكر بذكاء مثل الإنسان وأن يـ handle situations جديدة هو فقط ينفذ ما تم برمجته عليه

(3) Replacement of Labour in a populated place
استبدال العامل البشري بال robot تؤدي إلى عمل أزمة بطالة unemployment وخصوصاً في البلاد التي تعداد سكانها كبير

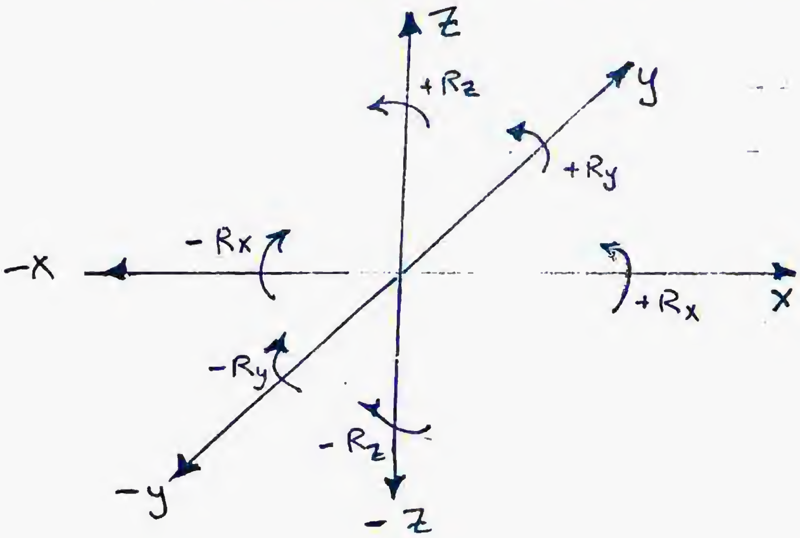
(4) Real time Response
سرعة استجابته بطيئة.

5

Degree of freedom (DoF) اتجاهات حركة ال Robot

a body suspended in Space can have :

- Six positive degree of freedom
  - على المحاور - 3 translatory or Linear movement along +ve axis
  - حركه دورانية حول المحاور - 3 rotatory anticlockwise movements
- Six negative degree of freedom
  - 3 translatory or Linear movement along -ve axis
  - 3 rotatory clockwise movements

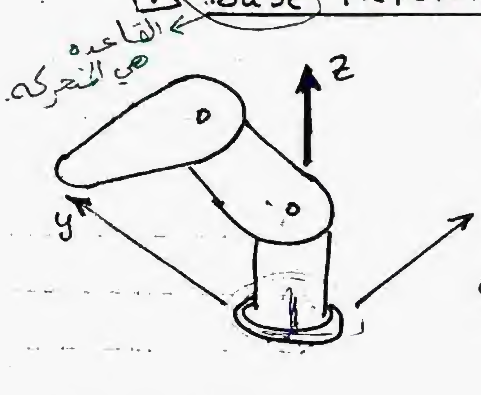


3 4

The Reference Frames

يوجد ثلاث أنواع من ال Frames الخاصة بحركة ال Robot :

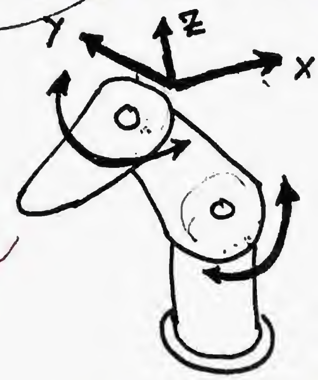
1. Base Reference Frames



لو فرضنا ان x,y,z هم ال axes للقاعدة (Base). فإن هذا النوع تكون به القاعدة اما ثابتة (fixed) أو (rotate) تدور حول x-axis. حسب ال application.

assum that x,y,z are axes for Base then this Type the Base is fixed or rotate around x-axis, that Depend on the application.

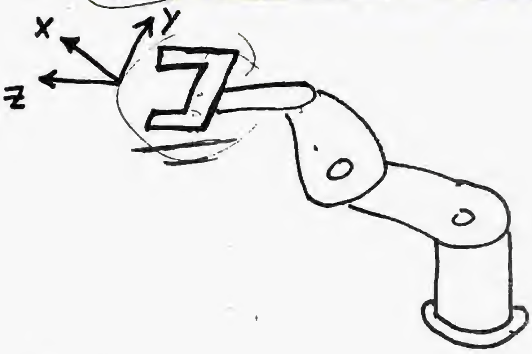
2 Joint Reference Frame



ال reference axes معرفة عند ال Joints .
و ال Joint يمكن أن يتحرك
حركة دورانية (Rotational) أو حركة
خطية (Translatory) .

x, y, z are reference axes for joints and joint could move Rotational movemet or Translatory movement.

3 Tool Reference frame



ال axes معرفة عند arm tip
أو ال Robot hand

x, y, z are reference axes at arm tip or Robot hand.

Robot joints

- هذه ال Joints مصنوعة لعمل حركة طولية (Linear) أو دورانية (rotary) أو كروية (spherical).
- المسميات
  1. prismatic joints - لعمل حركة طولية
  2. revolute joints - لعمل حركة دورانية
  3. spherical joints - لعمل أكثر من حركة دورانية (كروية)

Prismatic joints for linear movement ⟶ link
revolute joints for rotary movement ⟶ joint (Rollmann)
spherical joints for spherical movement

(3) 11-22 (3)

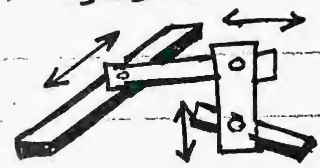
Robot Configurations

Workspace
scale

أنواع الحركات التي يقدم بها الـ robot

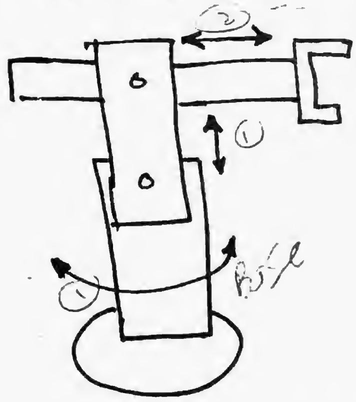
[1] Cartesian Robot (3P)

يحتوي على 3 prismatic joints و التي تعطي 3 حركات طولية (Linear) في اتجاه x ، y ، z .



[2] Cylindrical (2PR) Robot

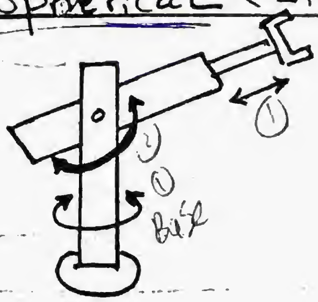
يحتوي على 2 primatic joints و التي تعطي حركتين طوليتين حول 2 axes و ايضاً يحتوي على 1 revolute joint تعطي حركة دورانية



polar

[3] Spherical (2RP) Robot - Articulated

2 Revolute joints
1 prismatic joint

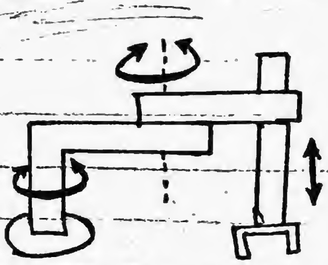


comprised type

الوحيد المتاح

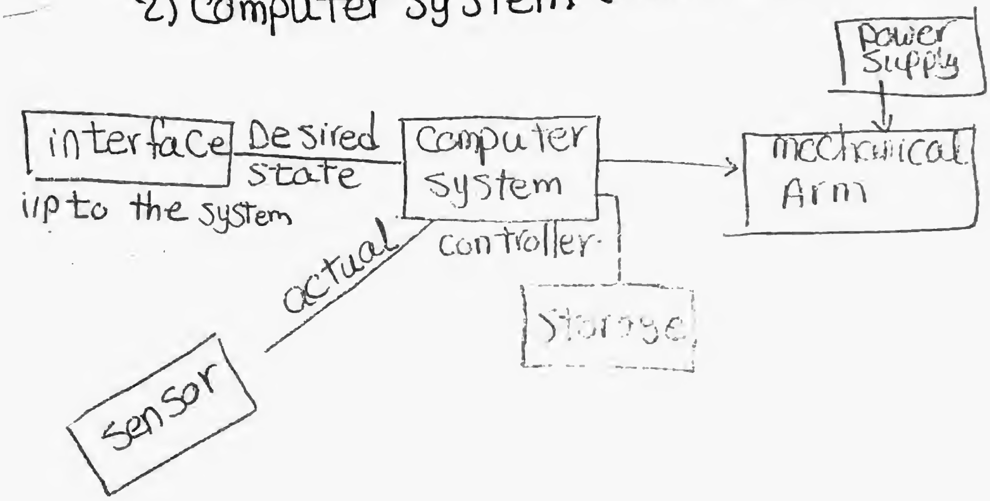
[4] SCARA (Selective Compliance Assembly Robot Arm)

* 2 horizontal parallel Revolute joints
* 1 vertical prismatic joint.



Components of Robotics System:-

1) Mechanical ARM.
2) Computer System (Controller).



①

Robot Components

manipulator
1) Links → Rigid body
2) Joints → Connections between 2 links
• Revolute Joint (θ) position بتغير الزاويه و ال
• prismatic Joint (d) بتغير الاستقالـه
3) E.E

1 Arms

ARM = Robot = Link + Joint ⇒ also Called Manipulator

2 End-effectors (Gripper, tool, Hand)

part Connected to the end of the arm Making the robot hand

3 Actuators

produce, rotary, translatory movement in Links.
هى المصدر الرئيسى للحركة فى ال Joints.

actuators →
- hydraulic
- pneumatic
- servo motors
- stepper motors

4 Sensors

* تحدد مكان (position) لل Robot
* تجمع بيانات عن البيئة المحيطة (Surrounding Environment).
* أمثلة Speed Sensor, Force sensor

5 Controllers

هو الجهاز الذى يدخل إليه الإشارات القادمة من Sensors و يقوم بمعالجتها و إخراج اشارة إلى ال actuator لإنشاء Action.

Sensor → Controller → actuator

6 Software and Hardware

a Computer SW & HW process through Computation to give the needed position, Speed, and accuracy of motion

• Robot Components

For an industrial robot to carry out the assigned task of its capability, it has to have certain parts and accessories as listed and explained below.

1. Arms. The various links and joints make the anatomy of the robot which are also known as manipulators. Manipulators are the mechanical parts of a robot.

2. End-effectors. The part that is connected to the end of the arm constituting the robot hand is the end-effector which in itself is different for different applications, like spray coat gun, a welding electrode hodler, part gripper, glue applying device or a special purpose tool.

3. Actuators. The joints of the robots are powered by what are known as actuators, that produce, rotary or translatory movement in the links. The power delivering systems can be hydraulic or pneumative drives, and servo-motors or stepper motors which are direct drive types.

4. Sensors. These are the parts that recognise the robots position while in movements or when static. The further movements or action depend on the feedback of the information collected by the sensors. Sensors also perform the function of gathering data about it surrounding (work cell) which aids in processing the task. The touch and tactile sensors, vision system, force sensors, speech processors are some of the examples to the sensors.

5. Controllers. These are electronic devices which manipulate the signals from the sensors, to be provided to the drives in a understandable pattern to produce actions. The action produced may be matching or may not be tallying with the desired output. The deviation is fed back in the form of an error which adjusts the reference input to a actuating signal. The control elements and the feed back elements constitute the control system.

6. Software and Hardware. A computer software and hardware process through computation of the symbolic codes to derive the needed purpose of position, speed and accuracy of motion obtained by the kinematic equations. The monitors the peripherals and computer systems are the hardware parts. The programming languages can be a low level language like machine language or a high level language like the present day high-level languages.

The robots can be made out of the above mentioned components designed and selected to suit the derived specifications to fulfill the needs of a particular industrial tasks of material handling, welding, painting, gluing and assembly tasks, in spite of that the present day robots copy the functions and actions performed by the human beings.

9

• Robot Specifications (22)

Any standard product has to be designed and marketed under certain requisite specifications or characteristics which aids in making decision to select and categorize it. Definite specifications that an industrial robot should bear are the maximum load carrying capacity, the repeatability or accuracy, the precision and the maximum and minimum reach defining the work space.

+ Payload : The rated load carrying capacity of an industrial robot is defined by its weight of the object or the tool held by the gripper, without affecting other functional characteristics like allowed tip deflection, control of motion along defined path etc. The overload may lead to the malfunctioning of the robot systems.

+ Repeatability : The accuracy with which the particular defined position can be repeatedly achieved by a robot is the repeatability. To arrive at the repeatability of a robot the statistical procedure of distribution of the positions have to be recorded and analysed and the estimated error has to be adjusted through programming the repeatability is affected by the

condition of the robot components also. The error in robot positions can give a random picture also, which has to be defined by the experimentations.

+ Precision : The reach of a position of a robot is defined by the resolution of the actuators and the control feed back systems. The robots precision is given in length units.

+ Reach : The lengths of the lengths of the links the configurations define the reach of an industrial robot. The maximum and the minimum extents of the robot positions give an idea about the reach of the robot, which is also useful in the specification of the work-envelope of the robot.

• Modes of Programming and Control

The instructions for path and position controls are provided through different programming languages for different robots manufactured by different companies. The instructions are the codes which vary from low level machine languages to high level languages understandable easily. The optimal paths to be followed to achieve the desired positions through the control system actions are coded and recorded in the form of software. In the process of development the following robot languages are generated for the purpose of programming.

APT — Automatically Programmed Tooling.
AL — Assembly Language
VAL — Victor's Assembly Language
AML — A Manufacturing Language
MCL — Manufacturing Control Language.
AUTO PASS — An IBM Language.

1.11 THE CHARACTERISTICS AND APPLICATION OF THE PRESENT ROBOTS (INDUSTRIAL)

Features → / ← Application	Degree of Freedom	Structure	Drive System	Program	Control System
Material Handling	3–5	Jointed arm	Pneumatic or Hydraulic	Manual or Powered lead through	Limited sequence or point-to-point playback
Machine Loading and Unloading	4–5	Polar, Cylindrical, Jointed arm	Electric or Hydraulic for (Heavy pay loads)	Powered lead through	Limited sequence or point-to-point playback
Spot Welding	5–6	Polar, Jointed arm	Hydraulic or Electric (light)	Powered lead through	Point-to-point playback
Arc Welding	5–6	Polar, Cartesian, Jointed arm	Electric or Hydraulic	Manual or Powered lead through	Continuous path playback
Spray Painting	6 or more	Jointed arm	Hydraulic	Manual lead through	Continuous path playback
Assembly line	3–6	Jointed arm, Cartesian, SCARA.	Electric	Powered lead through or textual language	Point-to-point or continuous.

1.12 ADVANCED TECHNOLOGICAL FEATURES OF A MODERN ROBOTS

- Multiple adaptable robotic arms with modular construction.
- Multiple nodes with one controller assisted by temperature gauge, pressure gauge and position sensors.
- **Motion Control :** By mechanical couplings (coupled motion control) and co ordinated kinematics and dynamics.
- Large work envelopes and higher payloads managed and controlled with servo tuning to avoid resonance and vibration.
- Usage of micro-controllers and embedded systems for less power requirement, compact in size, changeable functions, less movable parts for longer life, chip forming the brain.
- Controller Area Network (CAN) connections.
  - — controls efficiently the distributed intelligence.
  - — good price performance ratio.
  - — give reliability through error detection and error handling system.
  - — with immunity against electromagnetic interference.
  - — giving dynamic connection and disconnection of nodes for flexibility.
  - — providing real time capability for better repeatability, accuracy and precision.
  - — **Communication :** Radio frequency and infra red links for digital communications.
- Programmable Automation Control (PAC) for rapid advancement in capability for which re-engineering is needed, good portability of control engine.
- **Robot Vision :** Machine vision replaces human vision through video cameras, special computer hardware and software.

1.13 NEED FOR ROBOTS

- Accuracy aspect : The robots can perform tasks with highest accuracy, repeatability and the finish is of high quality.
- Environmental aspect : They can operate under the environments hazardous to human being.
- Human aspect : The human error is eliminated by use of robot. Human beings cannot work round the clock without fatigue.
- Skill aspect : The robots controlled by computer program can execute the tasks with better skill than human being.
- Performance aspect : Productivity is enhanced by induction of robots. They can produce better performance and efficiency than human being.
- Automation aspect : The highest technology component of automation in robots can give a competitive edge in the international level.

1.14 THE CHARACTERISTICS AND APPLICATIONS OF FUTURE INDUSTRIAL ROBOT

Features → / ↓ Application	Degree of Freedom	Structure	Drive System	Program	Nature of Task	Control System
• Material handling	3–5	Jointed adaptable robot arm	Servo motors	Programmable automation control (PAC)	Safe/hazardous complicated	Motion controllers with sensor technology.
• Part loading and unloading	4–5 Multiple arms	Polar, cylindrical, Jointed arm (Adaptable)	Electronic, Servo motors (For heavy payloads)	Programmable automation control (PAC)	Complicated and safe environs.	Micro controllers and Motion controllers with vision.
• Spot (Tack) Welding	5–6	Polar. Jointed adaptable robotic arm	Electronic stepper Motors.	Programmable Logic controllers (PLC)	Simple and safe.	Micro controllers with changeable functions.
• Arc Welding	5–6	Polar, modular cartesian with adaptable jointed arm.	Direct drive servo motors	Programmable automation, control (PAC)	Complicated and unsafe.	Continuous path motion controllers with sensor technology.
• Spray Coating	6 or more	Jointed arm with adaptable gun	Hydraulic actuators	Programmable Logic Controllers (PLC)	Simple and unsafe.	Continuous path motion controllers.
• Electronic Assembly	3–6 Multiple arms coupled motion.	Jointed adaptable, cartesian modular robotic arm.	Stepper motors and direct drives	Programmable Automation Control with Controller area Network (CAN)	Complicated and safe.	Micro controllers, nodes with sensors and end effectors with vision.

Application of Robots :

1. **Material handling.** The jointed arm robots with 3–5 degrees of freedom can serve the material handling application. Hydraulic or pneumatic drive with manual or powered lead through teaching would give motion in the present robot designs. The next generation robots are expected to use servo motors with Programmable Automation Control (PAC). In the future robot motion controllers with sensor technology would replace the point-to-point and sequence control action. They are expected to be used in both safe and hazardous environments.
2. **Machine loading and unloading robots.** Polar (P2R) robots, cylindrical (2PR) robots jointed arm (3P) robots with 4–5 degrees of freedom are used for such application. Electronic and servo drives are the future trends in the drives as compared to present electrical and hydraulic drives. PAC can replace the powered

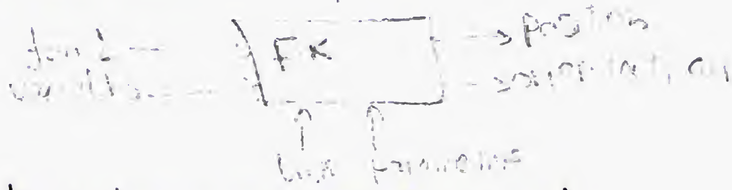
13) Define Robot kinematics.

Description of motion of the robot without Consideration of the force and Tourque Causing the motion.

geometric Description.

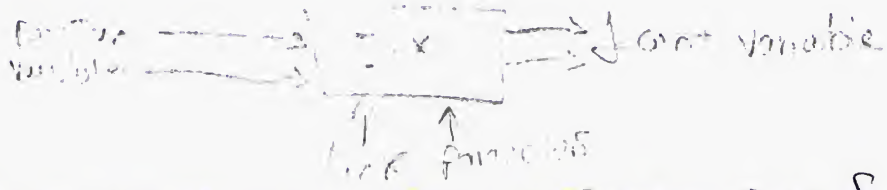
14) Robot F.K (give feedBack about E-E)

~~Description~~ Determination of the (actual) position and orienTation of the E-E, given the values for joint variables of the robot.



Robot I.K (determine Contrale action)

Determination of the values of the joint variable of the robot given the (desired) Position and orienTation of E E



15) Define the 2 DH assumptions for frame assignement in F.K

DH convention → used for selecting reference frame of robot

6 parameters → reduced to 4

F ال3 عوامل

a_i → link length

y_i → $\perp$ z_{i-1}

x_i → $\not\perp$

Cordinate frame assumption

① DH ⊥ → Cordinate frame is prependicular to the axis Z_{i-1}

→ DH1 → axis x_i is prependicular to the Z_{i-1}

→ DH2 → axis x_i is intersects to the Z_{i+1}

assumption H_i is achived by

② under the above assumption

① RoT

② TRA

③ T

④

Homogeneous Transformation

الـ Rotation Matrix التى تم شرحها سابقاً" لا تتضمن حركة Linear هى فقط تعبر عن الحركة الـ Rotational ولأخذ الحركة الـ Linear فى الاعتبار يجب أن يكون الـ position vector للنقطة p بدل من 3x1 أن يكون 4x1 وتصبح الـ Matrix كما يلى

النقطة إلى بتحركها

$$\text{Homogeneous Transformation Matrix} = \left[\begin{array}{c|c} \text{Rotation Matrix } (3\times3) & \text{position vector } (3\times1) \\ \hline \text{prespective Transformation } (1\times3) & \text{scale factor } (1\times1) \end{array}\right]$$

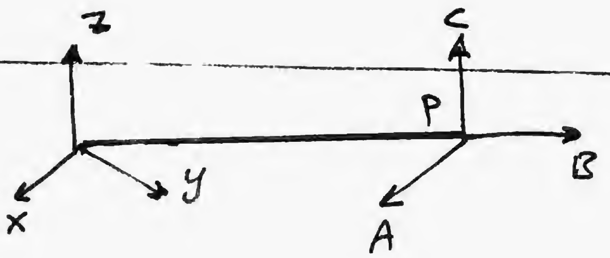
لوصف الحركتين الطوليه و الدورانيه

دائماً = 0   = 1

$$P_{xyz} = H \, P_{abc}$$

reference frame ← P_{xyz} ; P_{abc} → Mobile frame

		c → cos s → sin
Homogeneoust Rotation ation about OX-axis	$H(x,\alpha)$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Homogeneous Rotation about OY-axis.	$H(y,\phi)$	$\begin{bmatrix} c\phi & 0 & s\phi & 0 \\ 0 & 1 & 0 & 0 \\ -s\phi & 0 & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Homogeneous Rotation about OZ-axis	$H(z,\theta)$	$\begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Homogeneous translation by a vector P_{xyz} حركه الخطيه	$H_{(trans)}$	$\begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 4



$$\boxed{\begin{aligned} P_{xyz} &= H\ P_{abc} \\ P_{abc} &= H^{-1}\ P_{xyz} \end{aligned}}$$

How Can we get H^{-1}

$$H = \left[\begin{array}{c|c} R\,(3\times3) & \underset{(3\times1)}{P} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

$$H^{-1} = \left[\begin{array}{c|c} R^T & -R^T P \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} C\theta & -S\theta & 0 & P_x \\ S\theta & C\theta & 0 & P_y \\ 0 & 0 & 1 & P_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = H(z,\theta)$$ يعني لو كان

cos -sin
sin cos

$-R^T P$

cos +sin
-sin ...

$$\left[\begin{array}{ccc|c} C\theta & S\theta & 0 & -(P_x C\theta + P_y S\theta) \\ -S\theta & C\theta & 0 & -(-P_x S\theta + P_y C\theta) \\ 0 & 0 & 1 & -P_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = H^{-1}(z,\theta)$$ فان

$-P_x$ $\quad -R^T P$
$-P_y$
$-P_z$

cos -sin
sin cos

$-P_x$
P_y
P_z

Ch.3

Robot Motion Analysis

Link - Joint - EE

manipulator = ARM = Joints + Links

Mechanical Manipulator

هى دراسة الحركة النسبية (relative motion) والتى تغطى العلاقة

بين الـ objects وبعضها وبين الـ objects والـ Manipulator elements

علشان اوصل الـ EE لمكان معين معروف لحد الـ position للـ object والـ orientation بتاعي

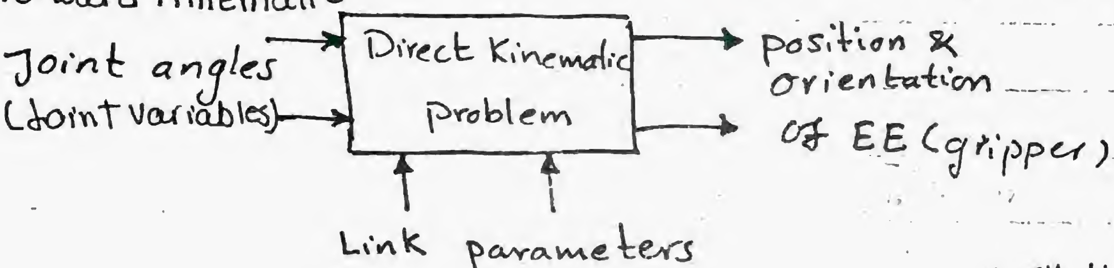
Robot Arm Kinematics

علم الحركة

EE pose = position + orientation

وصف تحليلى للحركة الهندسية للـ manipulator بالنسبة لنظام الإحداثيات للـ base frame.

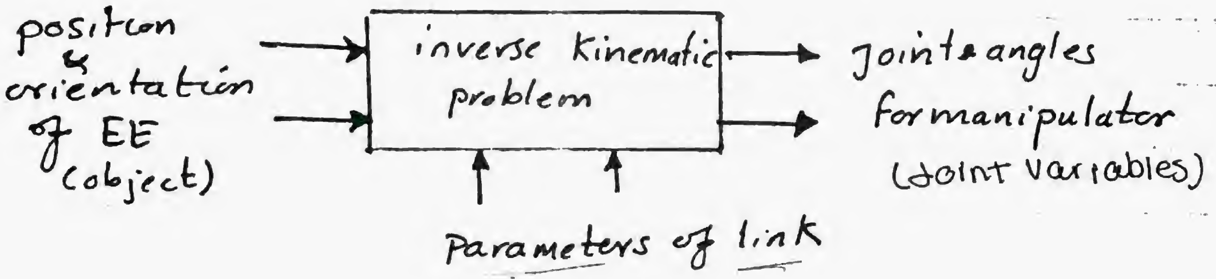
1) forward Kinematic



بندرس الايجاد دون النظر الى القوى

كل 2 link بينهم motor هو اللى شايل الـ object بناءً على torque معين

2) Reverse Kinematic



Transformations

- position vector

يمثل نقطة بالنسبة للإحداثيات الثلاثة.

عند عمل دوران حول أحد المحاور فإن المكان الجديد

للنقطة بالنسبة للـ base frame يكون معتمد على Rotation Matrix

assume

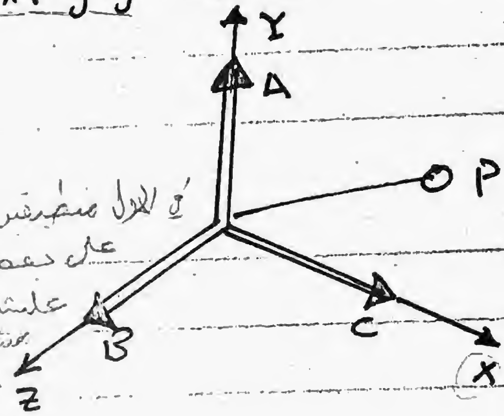
P : point

Assume $OABC \Longrightarrow$ rotated frame (mobailframe)

$OXYZ \Longrightarrow$ base frame

$$P_{ABC} = (P_a, P_b, P_c)^T = \begin{bmatrix} P_a \\ P_b \\ P_c \end{bmatrix} = P_a i_a + P_b i_b + P_c i_c$$

$$P_{xyz} = (P_x, P_y, P_z)^T = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = P_x i_x + P_y i_y + P_z i_z$$



في الاول منطبقين على بعض علشان مش في حركه

inverse of rotation matrix

$$\boxed{P_{ABC} = R^{-1} P_{xyz}}$$

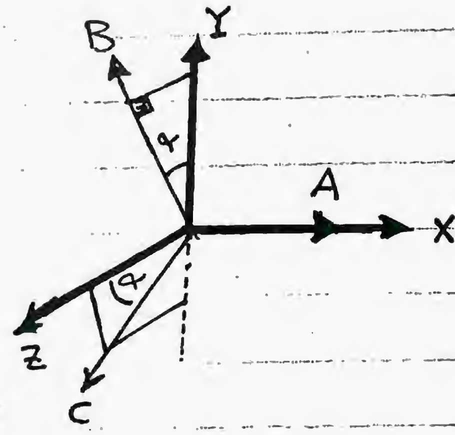
R → operator

R^{-1} → Rotation matrix

$$\boxed{P_{xyz} = R\, P_{ABC}}$$

cos sin
sin cos

Rotation about X-axis



$$P_x = P_A + 0(P_B) + 0(P_c)$$
$$P_y = 0(P_A) + P_B \cdot (\cos\alpha) + P_c(-\sin\alpha)$$
$$P_z = 0(P_A) + P_B(\sin\alpha) + P_c(\cos\alpha)$$

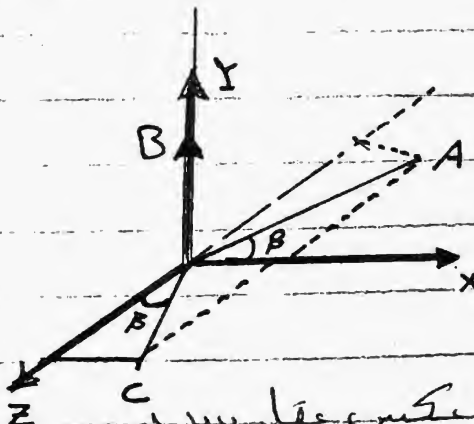
$$P_{xyz} = R(x, \alpha)\, P_{ABC}$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ P_c \end{bmatrix}$$

Rotation about Y-axis

$$P_{xyz} = R(y, \beta) \cdot P_{ABC}$$

$$\boxed{R(y,\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}}$$

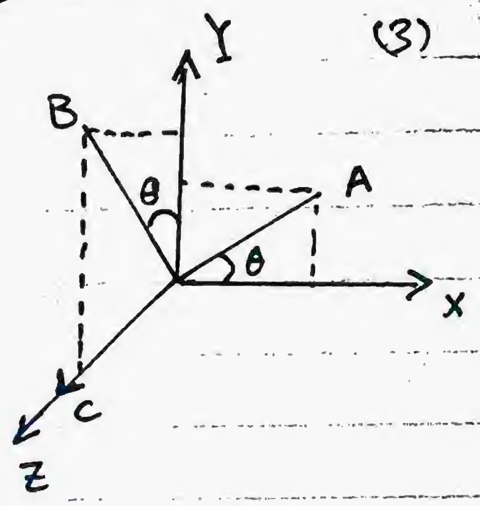


التدوير عكس عقارب الساعه تكون الزاويه موجبه و التدوير مع عقارب الساعه تكون الزاويه سالبه.

لو ماشي عكس عقارب الساعة تكون (-α)

(3)

Rotation about Z-Axis



$$P_{xyz} = R(z,\theta)\, P_{ABC}$$

$$R(z,\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the relation between the body attached frame (joints) with the base frame of reference is described by the Transformation Matrix

$$\left[\begin{array}{c|c} \text{Rotation Matrix } (3\times3) & \text{position vector } (3\times1) \\ \hline \text{perspective Transformation } (1\times3) & \text{stretching } (1\times1) \end{array}\right]_{4\times4}$$

Inverse Transformations

If the two coordinate frames are orthonormal (متعامده)

then $\boxed{X = RA}$

$$\boxed{A = R^{-1}X = R^T X}$$

تعني شروط التعامد

① $X_1^T Y_1 = 0 = Y_1^T X_1$

$Y_1^T Z_1 = 0 = Z_1^T Y_1$

$Z_1^T X_1 = 0 = X_1^T Z_1$

② $\|X_1\| + \|Y_1\| + \|Z_1\| = 1$

لو المحاور متعامده

وبالتالي يبقى الـ inverse

$R^{-1} = R^T$ (transpose)

Rotation	R matrix	$R^{-1} = R^T$
1. $R(x, \alpha)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$ ✓	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$
2. $R(y, \beta)$	$\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$	$\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$
3. $R(z, \theta)$	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P_{x,yz} = R\,P_{abc}$$

$$P_{abc} = \underset{R^{-1}}{R^T}\,P_{xyz}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos & -\sin \\ 0 & -\sin & \cos \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos & 0 & \sin \\ 0 & 1 & 0 \\ \sin & 0 & \cos \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos & -\sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R

$P_{\text{القديم}} = R\, P_{\text{الجديد}}$

$P_{\text{الجديد}} = R^{-1}\, P_{\text{القديم}}$

$$R(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos & -\sin \\ 0 & \sin & \cos \end{bmatrix}$$

$$R(y,\beta) = \begin{bmatrix} \cos & 0 & \sin \\ 0 & 1 & 0 \\ -\sin & 0 & \cos \end{bmatrix}$$

$$R(z,\theta) = \begin{bmatrix} \cos & -\sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problems

7

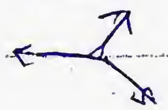
12, 13, 14, 15
16, 17, 18, 19

(4)

Problems

$P_{\text{القديم}} = R_z \, P_{\text{الجديد}}$

) the co-ordinate of a point $P_{abc} = (5,4,3)^T$ in the body co-ordinate frame OABC is rotated 30° about OZ axis. Determine the coordinates of the vector P_{xyz} with respect to base reference coordinate frame.



Solution

$$P_{abc} = (5,4,3)^T$$

$$\theta = 30^\circ$$

$$P_{xyz} = R(z,\theta)\,P_{abc}$$

$$= \begin{bmatrix} \cos(30) & -\sin(30) & 0 \\ \sin(3) & \cos(30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

$$= \underset{3\times3}{\begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} 2.33 \\ 5.964 \\ 3 \end{bmatrix}}$$

$$= [2.33,\ 5.964,\ 3]^T$$

---

② the coordinates of a point q_{abc} is given by $(7,5,3)^T$ which is rotated about the OX-axis of the reference frame OXYZ by angle of 60°. Determine the coordinates of the point q_{xyz}.

Solution

$q_{xyz} = ?$ $\quad q_{abc} = (7,5,3)^T$, $\alpha = 60$

$$q_{xyz} = R(x,60)\cdot q_{abc}$$

نفس الفكرة

③ the coordinates of a point P_{abc} in the mobile frame OABC is given by $[4, 3, 2\sqrt{3}]^T$.
if the frame OABC is rotated $\boxed{60^\circ}$ with respect to $\boxed{OY}$ of the OXYZ frame. find the coordinates of P_{xyz} with respect to the base frame.

$\boxed{\text{Solution}}$ $P_{abc} = [4, 3, 2\sqrt{3}]^T$, $\beta = 60$, P_{xyz} ??

$$P_{xyz} = R(y, \beta)\, P_{abc}$$

$$= \begin{bmatrix} 0.5 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2\sqrt{3} \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ -\sqrt{3} \end{bmatrix} = [5, 3, -\sqrt{3}]^T$$

---

④ for the following Rotation matrix determine the axis of rotation and the angle of the rotation.

$$R = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} \qquad R(y, \beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

بمقارنة هذه ال Matrix بال $R(y, \beta)$ نجد أنها تشبهها

$$\cos\beta = \frac{\sqrt{3}}{2} \quad , \quad \sin\beta = 0.5 \implies \tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{1}{\sqrt{3}}$$

$\boxed{\beta = 30^\circ}$ or $(\pi + 30^\circ) = 210$

$\beta = \tan^{-1}(1/\sqrt{3})$

لأن ليها نفس ال tan

الكيدي = الجسم

$y, 60$

a mobile body Referenced frame OABC is rotated $60°$ about OY-axis of the fixed base frame OXYZ. if $P_{xyz} = (2,4,6)^T$ and $q_{xyz} = (3,5,7)^T$ are the coordinates with respect of OXYZ plane, what are the corresponding coordinates of p and q with respect to OABC frame?

Solution

بدل ما عندنا نقطة لا، عندنا نقطتين . (نفس الكلام)

لأنه

$$P_{abc} = R(y,60)^{-1} P_{xyz} = R(y,60)^T P_{xyz} \rightarrow \text{orthognal}$$

$$q_{abc} = R(y,60)^{-1} q_{xyz} = R(y,60)^T q_{xyz}$$

$$R(y,60) = \begin{bmatrix} \cos 60 & 0 & \sin 60 \\ 0 & 1 & 0 \\ -\sin 60 & 0 & \cos 60 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.866 \\ 0 & 1 & 0 \\ -0.866 & 0 & 0.5 \end{bmatrix}$$

$$\boxed{R(y,60)^T} = \begin{bmatrix} 0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & 0.5 \end{bmatrix}$$

$$\boxed{P_{abc}} = \underbrace{\begin{bmatrix} 0.5 & 0 & -.866 \\ 0 & 1 & 0 \\ .866 & 0 & 0.5 \end{bmatrix}}_{R^{-1}} \underbrace{\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}}_{P_{x,y,z}} = \begin{bmatrix} -4.196 \\ 4 \\ 4.732 \end{bmatrix}$$

$$\boxed{q_{abc}} = \underbrace{\begin{bmatrix} 0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & 0.5 \end{bmatrix}}_{R^{-1}} \underbrace{\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}}_{P_{x,y,z}} = \begin{bmatrix} -4.562 \\ 5 \\ 6.098 \end{bmatrix}$$

(4)

~~6~~ the coordinates of point Q with respect to base reference frame is given by $[4, 2\sqrt{3}, 5]^T$. Determine the coordinates of Q with respect to abc mobile rotated frame of the robot if the angle of rotation with OX is $60°$.

Solution

$$Q_{abc} = R^{-1}(x,60) \cdot Q_{xyz}$$

متعامدين

$\because$ the two coordinate frame are orthogonal

$$R^{-1}(x,60) = R^T(x,60)$$

$$R(x,60) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 \\ 0 & \sin 60 & \cos 60 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 0.5 \end{bmatrix}$$

الصف يبقى عمود

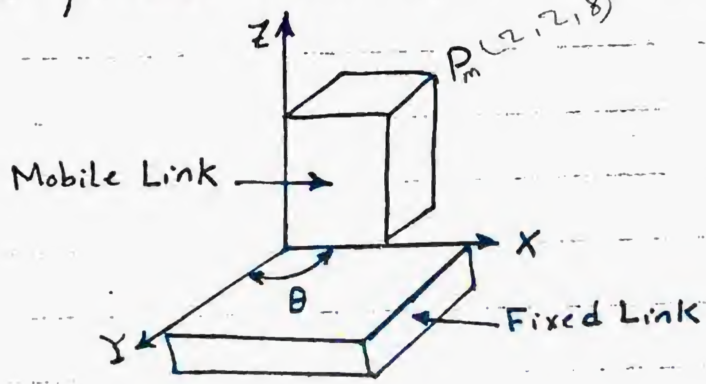
$$R^T(x,60) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & 0.5 \end{bmatrix}$$

$$Q_{abc} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 0.5 \end{bmatrix}}^{R^{-1}} \overbrace{\begin{bmatrix} 4 \\ 2\sqrt{3} \\ 5 \end{bmatrix}}^{Q_{x,y,z}} =$$

$$= [4, 6.06, -0.5]^T$$

cos

⑦ A single axis robot with a fixed base and a mobile link is as shown in the figure. Suppose the mobile frame has a point P_m given by $(2,2,8)^T$. Find the coordinates of the point P_F with respect to the base frame when $\theta_1 = 180°$ and $\theta_2 = 0°$



Solution

$P_M = [2,2,8]^T$ Coordinates of P in the mobile frame

$\theta_1 = 180°$, $\theta_2 = 0°$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) when $\boxed{\theta_1 = 180}$

$$R(z,\theta_1) = \begin{bmatrix} \cos 180° & -\sin 180° & 0 \\ \sin 180° & \cos 180° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(P_F)_{\theta=180°} = R(z,180°)\,P_m = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 8 \end{bmatrix}$$

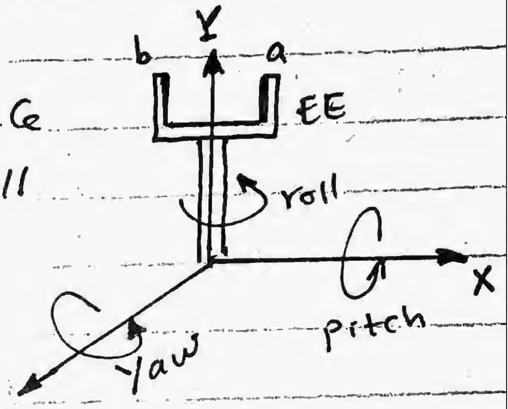
(ii) when $\boxed{\theta_2 = 0°}$

$$R(z,\theta_2) = \begin{bmatrix} \cos 0° & -\sin 0 & 0 \\ \sin\theta & \cos 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(P_F)_{\theta=0} = R(z,\theta_2)\,P_M = [2,2,8]^T$$

③

3) A robotic end effector is positioned as shown. the yaw of the end Effector (EE) is $\frac{\pi}{2}$ about Z-axis. the pitch of EE is π about X-axis and the roll of the EE is $-\frac{\pi}{2}$ about Y-axis.

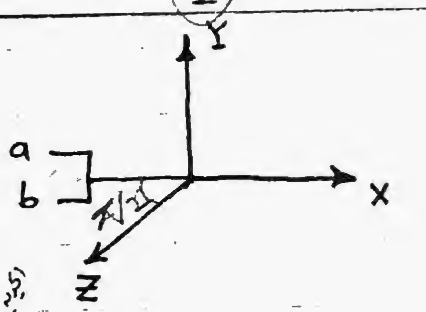
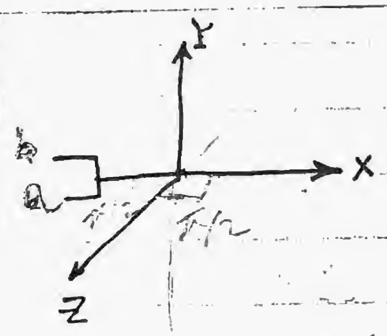
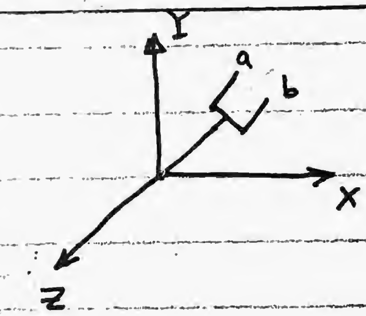


(a) Draw the sketch of EE in sequence after each of yaw, pitch, and roll motions.

ⓐ EE positions

Base ← Current →

$(Y, \frac{\pi}{2})(X, \pi)(Z, \frac{\pi}{2})\, I$

Yaw: $\frac{\pi}{2}$ about Z	Pitch: π about X	Roll: $-\frac{\pi}{2}$ about Y
		

ⓑ the Composite transformation matrix T which maps the Tip coordinates into the EE write Frame.

I R

ترتيبهم عكس السؤال

$$T = R(Y, -\tfrac{\pi}{2})\, R(\cancel{X}, \pi)\, R(Z, \pi/2)$$

$$= \begin{bmatrix} \cos -\frac{\pi}{2} & 0 & \sin(-\frac{\pi}{2}) \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos(-\frac{\pi}{2}) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi & -\sin\pi \\ 0 & \sin\pi & \cos\pi \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow \text{Composite transformation Matrix}$$

①

ⓒ Find the Coordinates of a point $P(0, 1.6, 0)$ at tool tip with ~~re~~ respect to wrist coordinate frame.

$$P_w = T.P_t = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1.6 \\ 0 \end{bmatrix} = [0, 0, -1.6]^T$$

problems

(10) A point $P_{abc} = (2,3,4)^T$ has to be translated through distance of +4 units along OX-axis and -2 units along OZ-axis. Determine the coordinates of the new point P_{xyz} by Homogeneous transformation.

Solution

$$P_{abc} = (2,3,4)^T$$

$$P_{abc} = (2,3,4,1)^T$$

المسافة التي تحركها

Homogeneous position vector $= (4, 0, -2, 1)^T$

$$H_{trans} = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} x \\ y \\ z \\ \end{array}$$

المسافة الي التحركها

$$P_{xyz} = H_{trans} \, P_{abc}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

احداثيات النقطة قبل الازاحه

$$= \begin{bmatrix} (1)(2) + 0 + 0 + (4)(1) \\ 0 + (1)(3) + 0 + 0 \\ 0 + 0 + (1)(4) + (-2)(1) \\ 0 + 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$P_{xyz} = [\underset{x}{6}, \underset{y}{3}, \underset{z}{2}]^T$$

في احداثي رابع بشايله.

(11) Determine the Homogeneous transformation matrix to represent a rotation of 30° about OX-axis and a translation of (8) units along the OB-axis of the mobile frame.

y

عندي ما

Solution $R(X_{,y-4})\ \text{I}\ \ R(X, 30°)\ \text{F}\ R(Z, 50°)$ 02H

$H(X, \alpha) = H(X, 30°)$ ← الدوران

$H(B, \beta) = H(B, 8)$ → الحركة في اتجاه B

$H = H(X, 30°)\ H(B, 8)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C30° & -S30° & 0 \\ 0 & S30° & C30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} a \\ b \\ c \\ \ \end{array} =$$

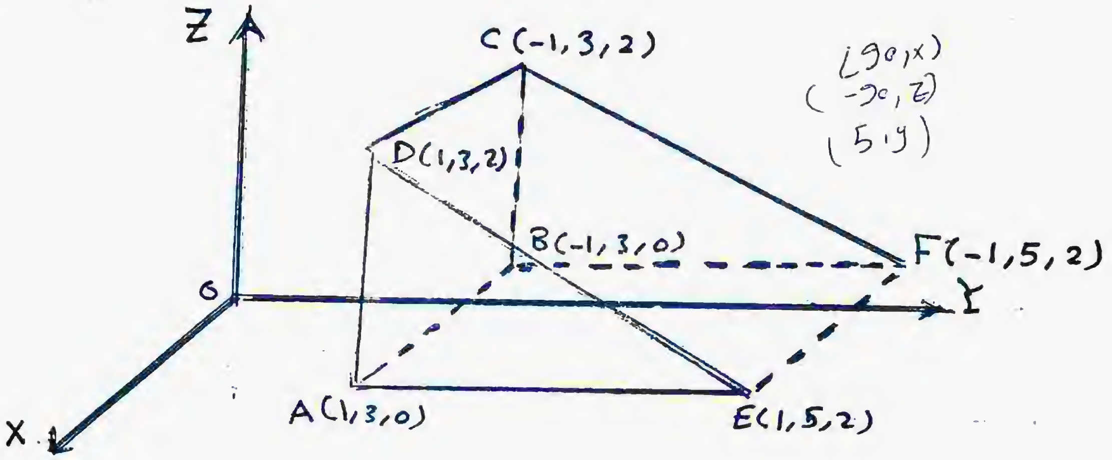
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -0.5 & 0 \\ 0 & 0.5 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -0.5 & 4\sqrt{3} \\ 0 & 0.5 & \frac{\sqrt{3}}{2} & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fixed fram ← عكس ترتيب السؤال. Backword

mobll fram → نفس ترتيب السؤال Forword

(12) a traingular prism with co-ordinates of its vertices indicated relative to the fixed reference frame OXYZ. is shown. the prism is moved to the new position with a rotation of +90° about x-axis, a rotation of -90° about Z-axis and a translation of 5 units in the y-direction.



Determine

(i) the homogeneous transformation describing the change in position of the prism.

(ii) the new coordinates of the vertices of the prism.

Solution

$$H(x,\alpha) = H(x,90)$$

$$H(z,\theta) = H(z,-90)$$

$$H_t(0,5,0)$$

$$H = \underbrace{H_t(0,5,0)\ H(z,-90)\ H(x,90)}_{\text{عكس ترتيب السؤال}}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & \cancel{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C(-90) & -S(-90) & 0 & 0 \\ S(-90) & C(-90) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(90) & -S(90) & 0 \\ 0 & +S(90) & C(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)

New coordinates of A $\qquad A = [H][a]$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

دانانێ بزووه

$A = (0, 4, 3)^T$

New Coordinates of B $\Rightarrow$ $B = [H][b]$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \\ 1 \end{bmatrix}$$

$B = [0, 6, 3]^T$

(13) Determine the homogeneous transformation matrix to represent the following sequence of operations

(i) Rotation of 60° OX-axis (X, 60)

(ii) Translation of 4 units along OX-axis (X, 4)

(iii) Translation of -6 units along OC-axis. (-6, C)

(iv) Rotation of 30° about OB-axis.

Solution

$$H(X,60) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H(X,4) = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H(C,-6) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H(B,30°) = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

fixed الثابت   المتحرك moving

عكس ترتيب المسألة   I   نفس ترتيبهم

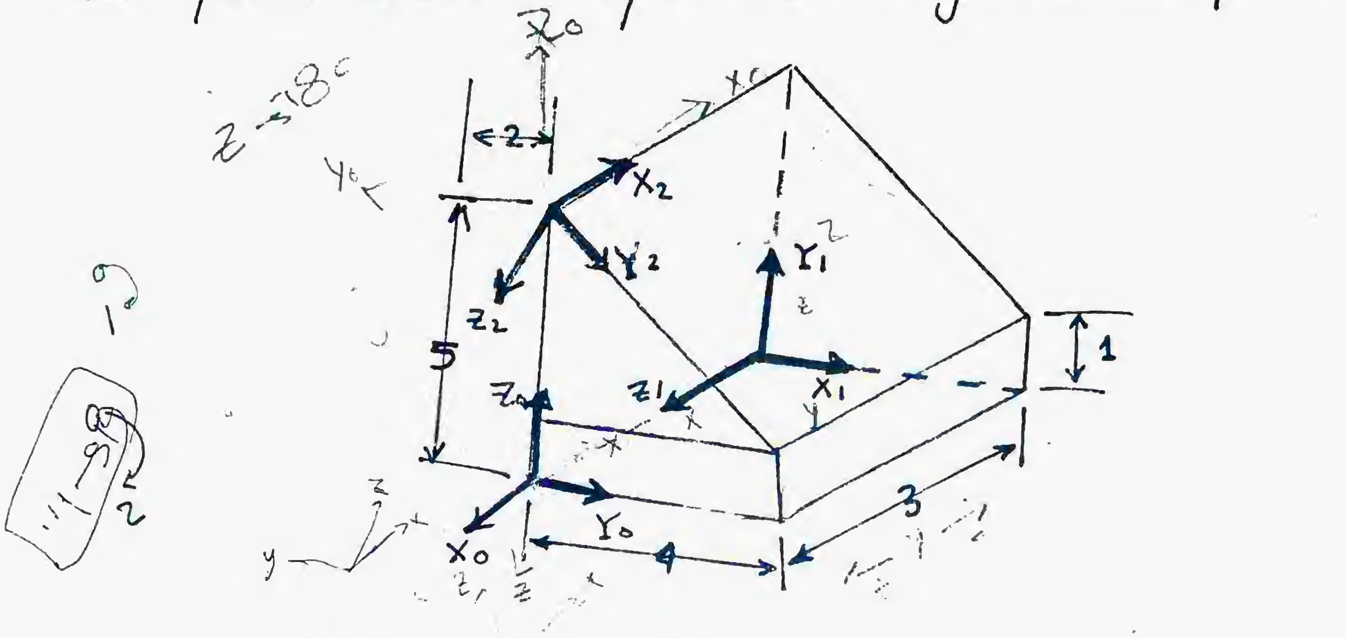
$$H = H(X,4)\, H(X,60°) \mid H(C,-6)\, H(B,30)°$$

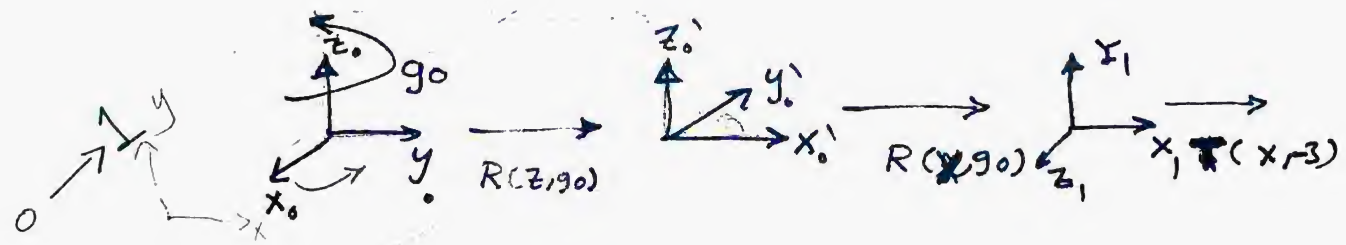
$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & \frac{\sqrt{3}}{2} & 0 \\ 0 & \sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

الترتيب أولاً ضع fixed frame معكوسين وبعدهم الـ mobile frame بالترتيب.

$$H = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & 4 \\ \frac{\sqrt{3}}{4} & 0.5 & -0.75 & -3\sqrt{3} \\ -0.25 & \sqrt{3}/2 & \sqrt{3}/4 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(19) for the object shown in fig. find 4x4 homogeneous transformation matrices $^{0}A_i$ for $i = 1, 2$ and thus find the $^{1}A_2$ i.e the transformation of frame at point 2 with respect to the frame at point 1

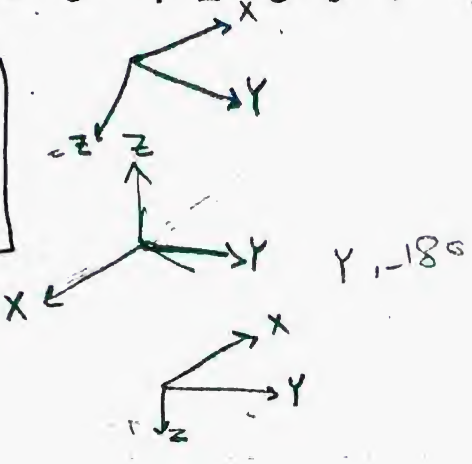




$$^{0}A_1 = H(-3,0,0)\,R(\cancel{X}_1,+90)\,R(Z_0,90)$$

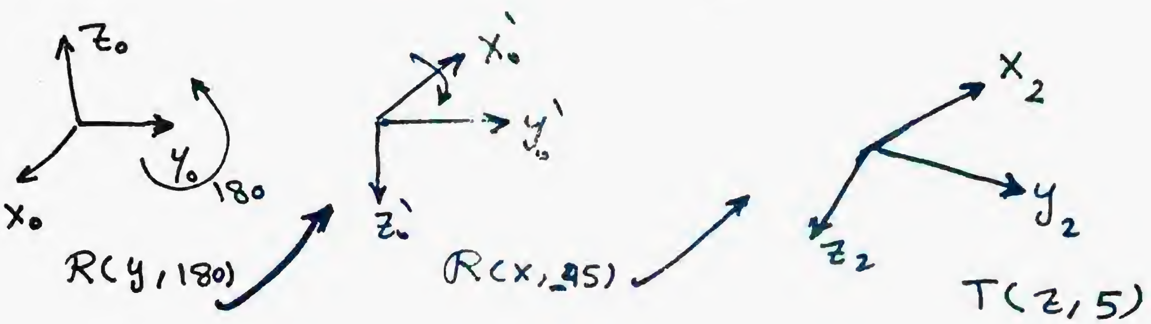
$$= \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \quad \end{bmatrix}$$



Transformation ${}^0A_2$ requires following operational sequence.

- Rotation by 180° about Y_0
- Rotation by -45 about x_0
- Translation of (0, 0, 5)



$${}^0A_2 = H(0,0,5)\,H(x_0;-45)\,H(Y_0,180^\circ)$$

$$= \begin{bmatrix} \ \end{bmatrix}\begin{bmatrix} \ \end{bmatrix}\begin{bmatrix} \ \end{bmatrix}$$

$$= \begin{bmatrix} \ \end{bmatrix}$$

(ii)

$${}^1A_2 = [{}^0A_1]^{-1}\,[{}^0A_2]$$

H^{-1}

${}^1A_2 = {}^0A_1$

${}^1A_2\ [{}^0A_1]^{-1}\,[{}^0A_2]$

$0 \rightarrow 1$, $0 \rightarrow 2$

$1 \rightarrow 2$

$0 \rightarrow 1 \quad 0 \rightarrow 2$

$1 \rightarrow 0 \quad 0 \rightarrow 2$

$${}^0A_2 = H_{trans}(0, 0, 5)\ R(Y_0, 180°)\ R(X_0, 45°).$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0A_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*ii*) To determine $\quad {}^1A_2 = [{}^0A_1]^{-1}\,[{}^0A_2]$

$${}^1A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans. $${}^1A_2 = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5 \\ -1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

15→19

Example 3.15. *Write down the homogeneous transformation matrices for the co-ordinate frames situated at the points A, B and C, with respect to base co-ordinate frame 0. What is the position and orientation of B with respect to frame 'C' ? Refer Fig. 3.15 for the considered co-ordinate frames.*

H

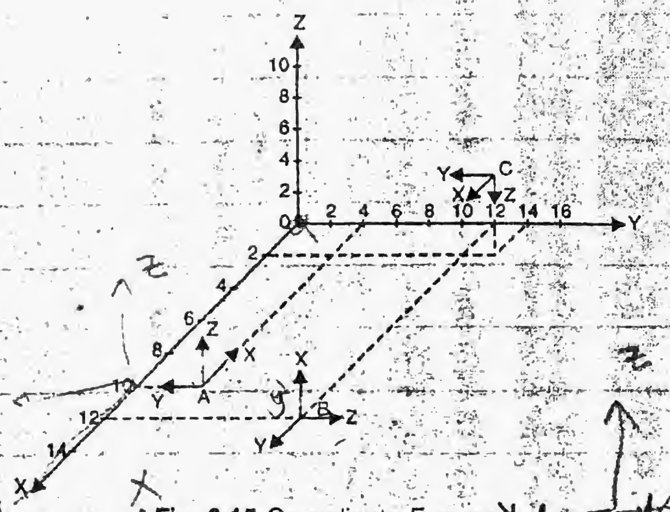
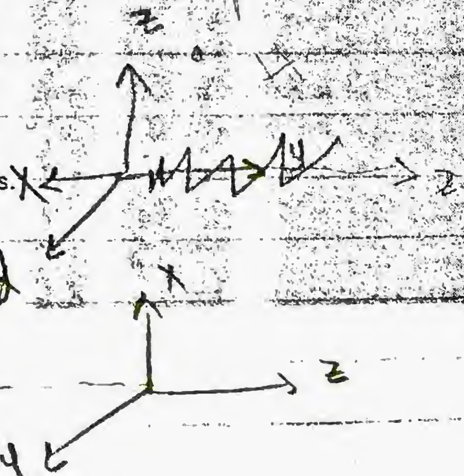
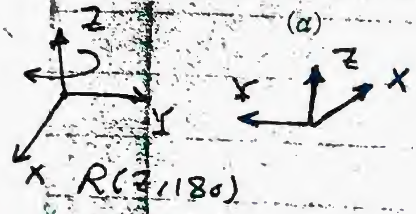


Fig. 3.15 Co-ordinate Frames.



Sol. (*i*) Homogeneous transformation matrices.

(*a*)

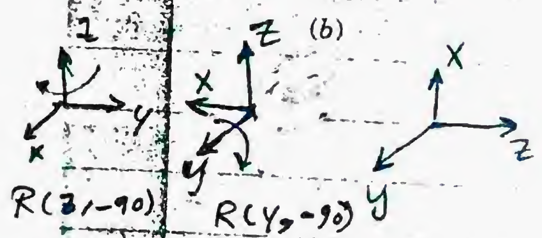


$$ {}^0H_A = H_{trans}(10, 4, 0)\, H(Z, 180°) I_4 $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 180 & -\sin 180 & 0 & 0 \\ \sin 180 & \cos 180 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(*b*)



$$ {}^0H_B = H_{trans}(12, 12, 0)\, H(Y, -90°)\, H(Z, -90)\, I_4 $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 12 \\ 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(*c*)

$$ {}^0H_C = H_{trans}(2, 14, 5)\, H(x, 180°)\, I_4 $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 14 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(*ii*) To determine the position of B w.r.t. C

$$ {}^CH_B = [{}^0H_C]^{-1}\, [{}^0H_B] $$

$$ [{}^0H_C]^{-1} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & +14 \\ 0 & 0 & -1 & +5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

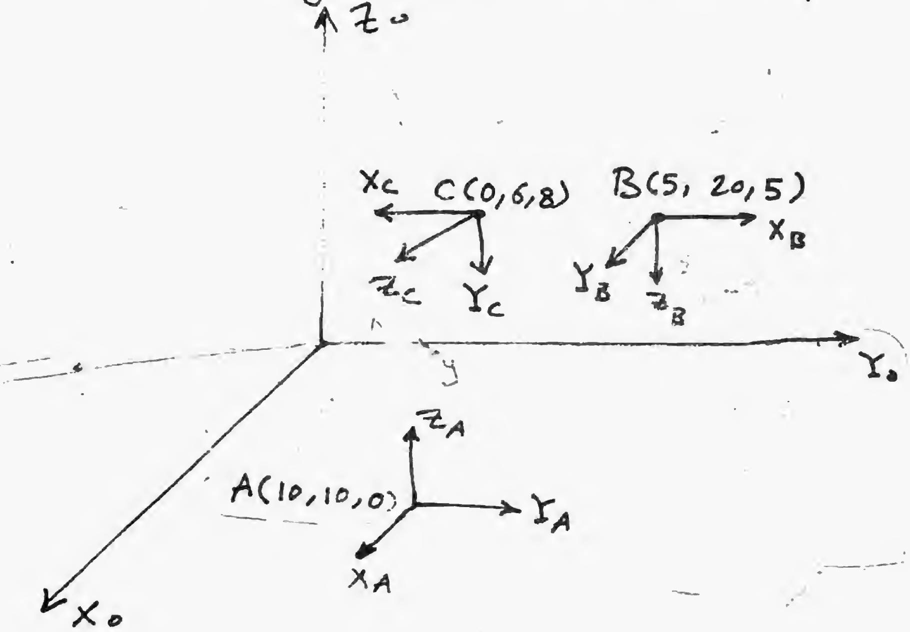
$$ \begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix} \qquad \begin{matrix} -2 \\ 14 \\ 5 \end{matrix} $$

$$ {}^CH_B = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & +14 \\ 0 & 0 & -1 & +5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 12 \\ 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 10 \\ -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The position vector = [10, 2, 5]^T

The orientation vectors = $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

(16) write down the homogeneous transformation matrices for the Coordinate frames Situated at the points A, B, and C with respect to $OX_oY_oZ_o$ frame. write down by inspection and matrix operation the position and orientation of frame B with respect to frame C.

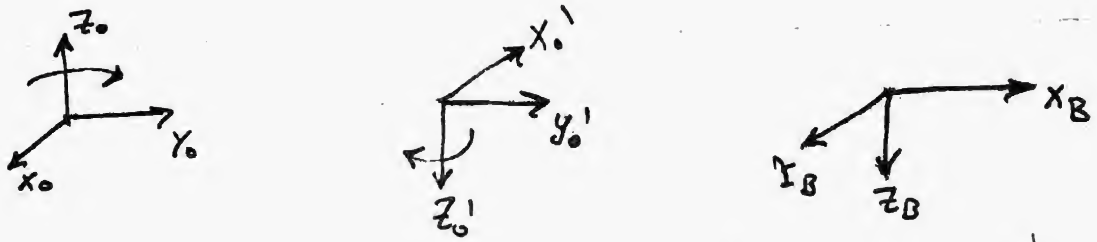


(a) homogeneous transformation matrix for point A.
operation to be performed $\Rightarrow T(10,10,0)$

في ازاحة فقط

$$H_A = H(10,10,0) = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Homogeneous transformation for point B
~~to~~ operation to be performed
- Rotation about Y_o by an angle +180
- Rotation about Z_o by an angle -90°
- Translation to position (5, 20, 5)



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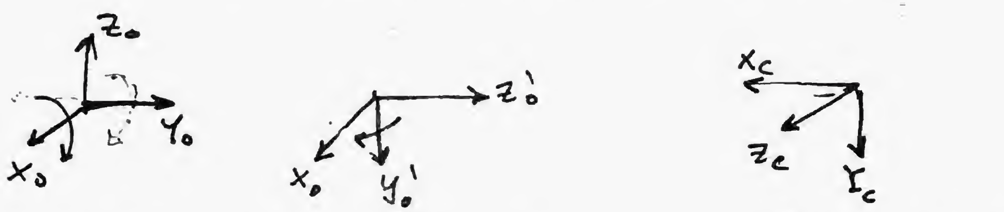
$$H_B = H_T(5,20,5)\,H(Y_0,180)\,H(Z_0,90)$$

=

(c) Homogeneous transformation matrix for C operations:

- Rotation of (−90°) about X_0
- Rotation of (−90°) about y_0
- Translation to (0, 6, 8)

$$H_C = H(0,6,8)\,H(y_0,-90)\,(X_0,-90)^{\mathrm{I}}$$



$R(X_0,-90)$ $\quad$ $R(y_0,-90)$ $\quad$ $T(0,6,8)$

H =

(d) position of B with Respect to C

$${}^{c}H_B = [{}^{o}H_c]^{-1}\,[{}^{o}H_B]$$

...work cell has a camera with in the setup. The origin of the six joint robot fixed to a base can be seen by the camera. A cube placed on the work cell-table is also seen by the camera. The homogeneous transformation matrix T_1 maps the camera with the cube centre. The origin of the base co-ordinate system as seen from the camera is represented by the homogeneous transformation system T_2.

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad H_2 = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) What is the position and orientation of the cube with respect to the base co-ordinate system ?

(b) After the system has been setup, some one rotates the camera 90° about the z-axis of the camera. What is the position and orientation of the camera with respect to robot's base co-ordinate system ?

(c) The same person rotated by 90° the object about the x-axis of the object and translated 5 units of distance along the rotated y-axis. What is the position and orientation of the object with respect to the robot's base co-ordinate system ?

Sol.

(a)
$$^{camera}H_{cube} = H_1 = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and
$$^{camera}H_{base} = H_2 = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is required to find $^{base}H_{cube}$. By 'chain product' rule

$$^{base}H_{cube} = {}^{base}H_{camera} \cdot {}^{camera}H_{cube} = (H_2)^{-1} H_1$$

$$= \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & -1 & 0 & -15 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 10 \\ -1 & 0 & 0 & -23 \\ 0 & 0 & 1 & -13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans. Position of cube is given by [10, – 23, – 13]

The orientation $[n, s, a] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Camera is rotated by 90° about the z-axis of the camera.

$$^{base}H_{camera} = (H_2)^{-1} . H(z, 90°)_{camera}$$

$$H(z, 90°)_{camera} = \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 0 \\ \sin 90° & \cos 90° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{base}H_{camera} = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & -1 & 0 & -15 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 8 \\ -1 & 0 & 0 & -15 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans. Position of the camera after the change is given by $[8, -15, -6]^T$

The orientation of camera with respect to base

$$[n, s, a] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(c) Now the object is rotated by 90° about x-axis of the object and translated by 5 unit distances along the rotated y-axis of the object.

$$^{b}H_{c} = {}^{base}H_{cube} . H(x, 90°) . H(y, 5)$$

i.e.,
$$^{b}H_{c} = \begin{bmatrix} 0 & 1 & 0 & 10 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 10 \\ -1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans. The position of the object with respect to the base = $[10, 7, 4]^T$

The orientation, $[n, s, a] = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

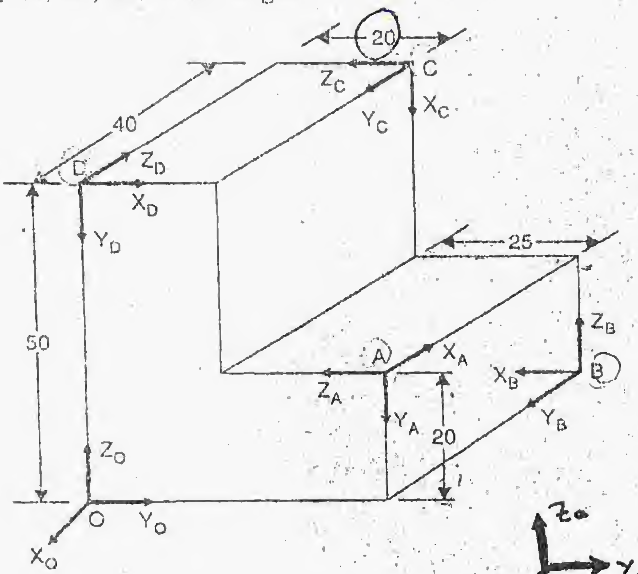
Example 3.18. *Write down the homogeneous transformation matrices for the co-ordinate frames attached to the corners A, B, C and D with respect to the base co-ordinate frame '0'. Also write down the transformation matrix for A with respect to 'C' frame and verify the same by finding the inverse.*

The object frame is as shown in Fig. 3.17. (VTU-Jan.Feb. 2004)

Sol. (*i*) Homogeneous transformation matrix for A.

The transformation involves following sequence of operation

- Rotation by –90° about X_0
- Rotation by 180° about Y_0
- Translation by (0, 45, 20) from origin



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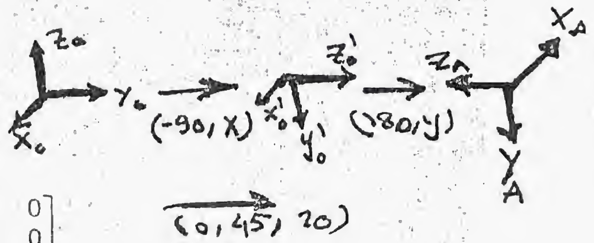


Fig. 3.17 Co-ordinate Frame.

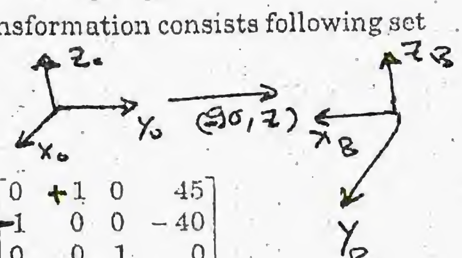
$$^0H_A = H_{trans}(0, 45, 20)\, R(Y_0, 180°)\, R(X_0, -90°)\, I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 45 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 45 \\ 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 45 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

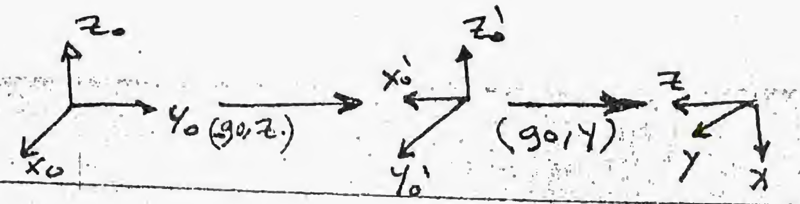
(*ii*) Homogeneous transformation matrix of B. The transformation consists following set of operations :

- Rotation by –90° about Z_0
- Translation by (45, – 40, 0)



$$^0H_B = H_{trans}(45, -40, 0)\, R(Z_0, -90°)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 45 \\ 0 & 1 & 0 & -40 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & +1 & 0 & 45 \\ -1 & 0 & 0 & -40 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(*iii*) Homogeneous transformation of matrix C transformation consists of following sequence of operations :

- Rotation of -90° about Z_0
- Rotation of (+90°) about Y
- Translation by (– 40, 20, 50)

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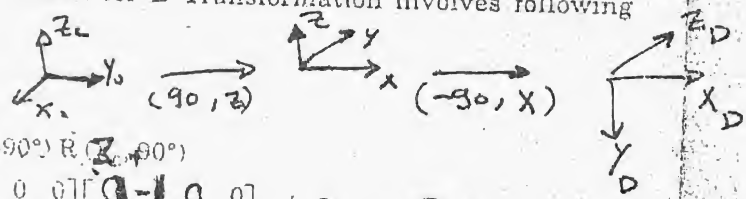
$$^0H_C = H_{trans}(-40, 20, 50)\ R(Z_0, -90°)\ R(Y_0, +90°)\ I_4$$

$$\begin{matrix} 0 & 0 & 1 & -40 \\ 0 & 1 & 0 & 20 \\ -1 & 0 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{matrix} \quad = \begin{bmatrix} 1 & 0 & 0 & -40 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -40 \\ -1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*iv*) Homogeneous transformation matrix for D Transformation involves following sequence of operations :

- Rotation by (+ 90°) about Z_0
- Rotation by -90° about X_0
- Translation of (0, 0, 50)



$$^0H_D = H_{trans}(0, 0, 50)\ R(X_0, -90°)\ R(Z_0, +90°)$$

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{matrix} \quad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous transformation A from w.r.t. C frame

$$^CH_A = [^CH_0][^0H_A] = [^0H_C]^{-1}[^0H_A]$$

$$= \begin{bmatrix} 0 & 1 & 0 & -20 \\ 0 & 0 & -1 & 50 \\ -1 & 0 & 0 & -40 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 45 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & +25 \\ 0 & 1 & 0 & 30 \\ 1 & 0 & 0 & -40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 3.19. *A six joint robotic manipulator equipped with a digital TV camera is capable of continuously monitoring the position and orientation of an object. The position and orientation of the object with respect to the camera is expressed by a matrix $[T_1]$, the origin of the robot's base co-ordinate with respect to the camera is given by $[T_2]$, and the position and orientation of the gripper with respect to the base co-ordinate frame is given by $[T_3]$.*

$$^C_O[T_1] = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^C_B[T_2] = \begin{bmatrix} 1 & 0 & 0 & -20 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & -1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad ^B_g[T_3] = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine

(*i*) *the position and orientation of the object with respect to the base co-ordinate.*

(*ii*) *the position and orientation of the object with respect to gripper.*

(VTU Jan.Feb. 2004 ; VTU May.June 2004)

Sol. Given :

$[T_1] = {}^{camera}T_{object}$
$[T_2] = {}^{camera}T_{base}$
$[T_3] = {}^{base}T_{gripper}$

T_1^{-1} O

C

$^b_O T \quad [T_2]^{2}\,[T_1] \quad {}^B H_O \quad [T_2]^{-1}\ {}^B_C\ {}^C_O$

0

b

(*i*) The position and orientation of object with respect to base co-ordinates, $[{}^{base}T_{object}]$
By chain product rule,

$$ {}^{base}T_{object} = {}^{base}T_{camera} \cdot {}^{camera}T_{object} = [T_2]^{-1}.[T_1] $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & -1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 25 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Ans. The position vector = $[25, 4, 2]^T$

$$ \text{The orientation matrix} = [n, s, a] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

(*ii*) To determine the position and orientation of the object with respect to gripper, by chain product rule,

$$ {}^{gripper}T_{object} = {}^{gripper}T_{base} \cdot {}^{base}T_{object} = [T_3]^{-1}(\underline{[T_2]^{-1}[T_1]}) $$

$$ = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 25 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

حسبناها قبل كدا

$$ {}^{gripper}T_{object} = \begin{bmatrix} 0 & 1 & 0 & (-8+25) \\ -1 & 0 & 0 & (4-6) \\ 0 & 0 & 1 & (-6+2) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Ans. The position vector = $[17, -2, -4]^T$

$$ \text{The orientation vectors, } [n, s, a] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. $$

3.13 MANIPULATOR PARAMETERS

A robot manipulator is a chain of rigid bodies, called links, connected in sequence by joints, known as lower pair joints. The links remain in contact at the joints with two surfaces sliding over one another relatively. There are totally six possible lower pair joints : prismatic (sliding) joint, revolute (rotary) joint, cylindrical, screw, spherical and planar joints. The robot manipulators are generally, designed with prismatic or/and revolute joints. In a serial open loop formation each link forms connection, at the most, with two other links. Each pair of a link and a joint contributes single degree of freedom. 'N' numbers of pairs provide 'N' degrees of freedom for a manipulator. Link 1 forms a joint '0' with the base which establishes an innertial co-ordinate frame for a dynamic system anylysis of a industrial robot. The last link at its free end accomodates a tool or a gripper. Both the base and the gripper are not considered as the part of a robotic manipulator.

In general the link '*k*' gets connected at the two ends with link $(k-1)$ and link $(k+1)$, forming two joints at the ends of connections. The link is characterised by the (*i*) distance (d_i) and (*ii*) the angle (θ_i), between the adjacent links. The joint is featured by (*a*) length (q_k) and (*b*) the twist angle (α_k) of the link *k*. The manipulator parameters determine structure and relative position of links in the arm.

Exams

Tanta University
Faculty of Engineering
Computers and control Dept. **Robotics**

4th year Computers
Midterm Apr.2012
Time : 1and half hr.

Answer all the following questions;

Question No. 1

1. Define a Robot.
2. Discuss the advantages and disadvantages of using robots in industry.
3. Compare hard automation with soft automation.
4. Discuss the impact of robotic induction on direct labour.
5. What are various types of reference frames attached to a robotic? Explain with example.
6. Briefly discuss the various robot components.

Question No. 2

1. The co-ordinates of a point P_{abc} in the mobile frame OABC is given by $[4,3,2]^T$. If the frame OABC is rotated 45° with respect to OY of the OXYZ frame, find the co-ordinates of P_{xyz} with respect to the base frame.

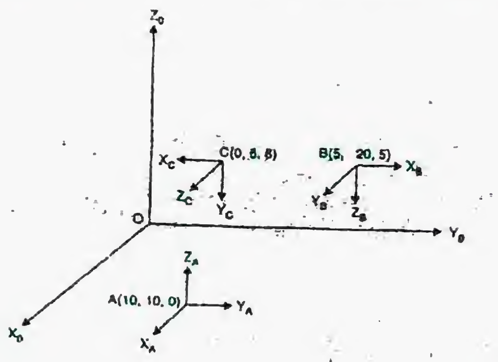
2. A mobile body reference frame OABC is rotated 30° about OY-axis of the fixed base reference frame OXYZ. If $P_{xyz} = [-2,4,6]^T$, $Q_{xyz} = [-1,3,5]^T$ are the co-ordinates with respect to OXYZ plane, what are the corresponding co-ordinates of P and Q with respect to OABC frame?

3. The co-ordinates of point Q with respect to base reference frame is given by $[4,2,5]^T$. Determine the co-ordinates of Q with respect to mobile rotated frame of the robot if the angle of rotation with the OX is 60°.

4. Determine the homogeneous transformation matrix to represent a rotation of 60° about OX-axis and a translation of 10 units along the OA-axis of the mobile frame.

5. Determine the homogeneous transformation matrix to represent the following sequence of operations:
   i. Rotation of 30° OX-axis.
   ii. Translation of 5 units along OX-axis.
   iii. Translation of -8 units along OB-axis
   iv. Rotation of 60° about OA-axis

6. Write down the homogeneous transformation matrices for the co-ordinate frames situated at the points A,B and C with respect to $OX_oY_oZ_o$ frame in the figure shown. Write down by inspection and matrix operation the position and orientation of frame B with respect to frame C.



problem 6. of queston No.2

Best wishes

Dr. Eng. Elasyed Sallam

Tanta University | Department: Computers and Control Engineering | Faculty of Engineering

Total Marks: 20 Marks

Course Title:Elective4(**Robotic**) اختياري 4 روبوتيك رابعة حاسبات Course Code: CCE4242 4th year

Date: 10.4.2013 (Mid-Term) Allowed time: 1 hour

Answer all the following questions;

Question No. 1

1. Define a Robot.
2. Discuss the advantages and disadvantages of using robots in industry.
3. Discuss the impact of robotic induction on direct labour.
4. What are various types of reference frames attached to a robotic? Explain with example.
5. Write the homogeneous transformation matrix of Euler I representation.

Question No. 2

1. The co-ordinates of a point P_{abc} in the mobile frame OABC is given by $[1,2,3]^T$. If the frame OABC is rotated 45° with respect to OY of the OXYZ frame, find the co-ordinates of P_{xyz} with respect to the base frame.

2. A mobile body reference frame OABC is rotated 30° about OX-axis of the fixed base reference frame OXYZ. If $P_{xyz} = [-1, 2, 3]^T$, $Q_{xyz} = [3, 2, 1]^T$ are the co-ordinates with respect to OXYZ plane, what are the corresponding co-ordinates of P and Q with respect to OABC frame?

3. The co-ordinates of point Q with respect to base reference frame is given by $[4,2,-1]^T$. Determine the co-ordinates of Q with respect to mobile rotated frame of the robot if the angle of rotation with the OX is 60°.

4. Determine the homogeneous transformation matrix to represent a rotation of 60° about OX-axis and a translation of 10 units along the OA-axis of the mobile frame.

5. Determine the homogeneous transformation matrix to represent the following sequence of operations:
   i. Rotation of 30° OX-axis.
   ii. Translation of 10 units along OZ-axis.
   iii. Translation of -5 units along OA-axis
   iv. Rotation of 60° about OB-axis

Best wishes ***Dr. Eng. Elasyed Sallam***

Course Title:Elective4(**Robotics**) Date: 10.4.2016 (Mid-Term)	اختياري ٤ روبوتيك رابعة حاسبات	Course Code: CCE4242 4th year Allowed time: 1 hour and half

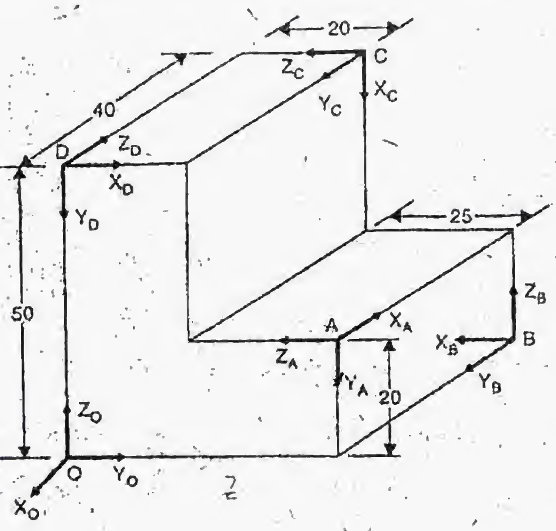
Answer all the following questions;

Question No. 1 (5 Degrees)

1. Define a Robot.
2. Compare hard automation with soft automation.
3. Discuss the impact of robotic induction on direct labour.
4. What are various types of reference frames attached to a robotic? Explain with example.
5. What are performance parameters? Define repeatability, resolution and accuracy.

Question No. 2 (15 Degrees)

1. A mobile body reference frame OABC is rotated 60° about OX-axis of the fixed base reference frame OXYZ. If $P_{xyz}=[2,4,6]^T$, $Q_{xyz}=[1,-3,5]^T$ are the co-ordinates with respect to OXYZ plane, what are the corresponding co-ordinates of P and Q with respect to OABC frame?
2. The co-ordinates of point E with respect to base reference frame is given by $[3,-2,5]^T$. Determine the co-ordinates of E with respect to mobile rotated frame of the robot if the angle of rotation with the OA is 30°.
3. Determine the homogeneous transformation matrix to represent a rotation of 30° about OY-axis and a translation of 10 units along the OB-axis of the mobile frame.
4. Determine the homogeneous transformation matrix to represent the following sequence of operations:
   i. Rotation of 30° OY-axis.
   ii. Translation of 10 units along OX-axis.
   iii. Translation of -8 units along OB-axis
   iv. Rotation of 60° about OB-axis
5. Write down the homogeneous transformation matrices for the co-ordinate frames attached to the corners A,B,C and D with respect to the base co-ordinate frame "0". Also write down the the transformation matrix for A with respect "C" frame and verify the same by finding the inverse.



Problem 5. of Question No.2

A
C A
Z → -90
y 90

Best wishes *Dr. Eng. Elasyed Sallam*

Course Title: **Robotic Systems** تخصصي ٤ روبوت رابعة حاسبات Course Code: CCE4242 4th year
Date: 4.6.2016 (Second term) Allowed time: 3 hrs No. of Pages: (2)

Answer all the following questions:

Question No. 1 *(20 marks)*

1. Discuss the advantages and disadvantages of using robots in industry.
2. What is workspace? Give the functional diagram with the workspace for the following robots i-3R-robot. ii-2RP robot.
3. Draw any two Euler angle systems and show rotations and angles.
4. What are performance parameters? Define repeatability, resolution and accuracy.
5. Compare hard automation with soft automation.
6. Define the term: Robot kinematics.
7. Differentiate between robot forward kinematics and robot inverse kinematics.
8. Mention the two DH assumptions for frame assignment in forward kinematics. Explain how they reduce the parameters required to relate frame i to frame $i-1$.
9. In your own words, explain briefly how machine learning can be used to estimate robot inverse kinematics. (Explain the steps of applying machine learning).

Question No. 2 *(20 marks)*

1. The co-ordinates of a point P_{abc} in the mobile frame OABC is given by $[2,4,5]^T$. If the frame OABC is rotated 45° with respect to OY of the OXYZ frame , find the co-ordinates of P_{xyz} with respect to the base frame.

2. A mobile body reference frame OABC is rotated 30° about OZ-axis of the fixed base reference frame OXYZ. If $P_{xyz}=[-1,2,3]^T$, $Q_{xyz}=[2,-3,1]^T$ are the co-ordinates with respect to OXYZ plane, what are the corresponding co-ordinates of P and Q with respect to OABC frame?

3. For the the object shown in figure 1, find the 4x4 homogeneous transformation matrices ^0A_i for i= 1,2 and thus find the transformation of frame at point 1 with respect to the frame at point 2 (i.e. $^2A_1$).

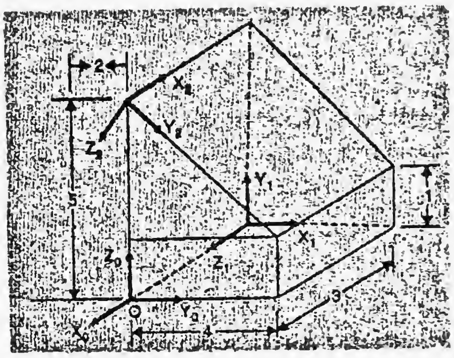


Figure 1
problem 3. of Question No.2

Question No. 3 *(22 marks)*

1. Determine the homogeneous transformation matrix to represent a rotation of 30° about OZ-axis and a translation of 20 units along the OB-axis of the mobile frame.

2. Determine the homogeneous transformation matrix to represent the following sequence of operations:

a. Rotation of 45° OZ-axis.
b. Translation of 4 units along OX-axis.
c. Translation of -4 units along OB-axis
d. Rotation of 90° about OA-axis

3. A robotic work cell has a camera with in the setup. The origin of the six joint robot fixed to a base can be seen by the camera. The homogeneous transformation matrix H_1 maps the camera with the cube centre. The origin of the base co-ordinate system as seen from the camera is represented by the homogeneous transformation matrix H_2.

$$\text{cube } H_1 \text{ camera} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{Camera } H_2 \text{ Base} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera H1 cube

a) What is the position and orientation of the cube with respect to the base co-ordinate system?
b) After the system has been setup, someone rotates the camera 90 about the x-axis of the camera. What is the position and orientation of the camera with respect to robot's base co-ordinate system?
c) The same person rotated by 90 the object about the z-axis of the object and translated 5 units of distance along the rotated y-axis. What is the position and orientation of the object with respect to the robot's base co-ordinate system?

Question No. 4

Base H cube   Camera H1 Base

(23 marks)

1. A six joint robotic manipulator equipped with a digital TV camera is capable of continuously monitoring the position and orientation of an object. The position and orientation of the object with respect to the camera is expressed by a matrix $[T_1]$, the origin of the robot's base co-ordinate with respect to the camera is given by $[T_2]$, and the position and orientation of the gripper with respect to the base co-ordinate frame is given by $[T_3]$. Where

$$T_1 = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2 = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } T_3 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine: i-the position and orientation of the object with respect to the base co-ordinate.
ii- the position and orientation of the object with respect to gripper.

2. For the Cylindrical manipulator shown in figure 2, Find the homogeneous transformation matrix describing the forward kinematics of the whole manipulator, i.e. the position and orientation of the end effector with respect to the base. (Hint: Apply DH-convention)

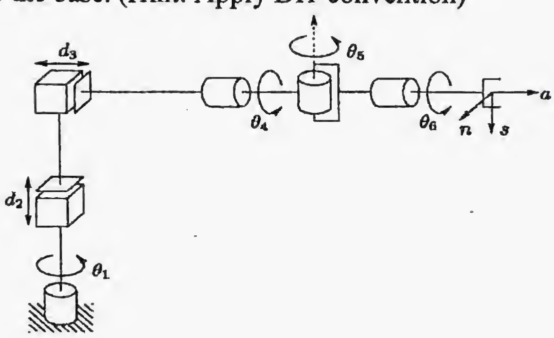


Figure 2
Problem 2. of Question No.4

Best wishes   ***Dr. Eng. Elasyed Sallam***